GAMMA

PREFACE

> If you are not comfortable with the concept of linearity (in mathematics),
> Take a look at the chapter Linear before continuing with this chapter.

A gamma is a transition "method" between a linear world (such as computers) and a non-linear world (such as display or human vision). It is known as "Transfer Function" by the most knowledgeable and learned individuals.

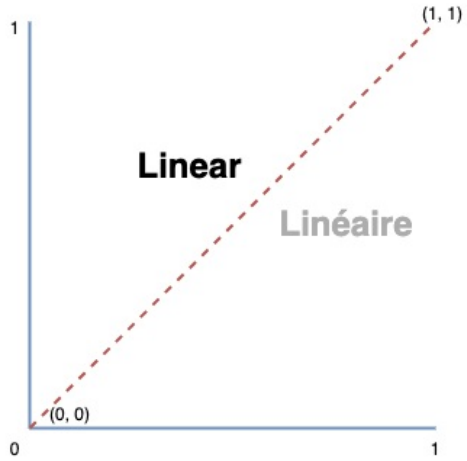
Visually, a gamma involves adjusting the luminance. By manipulating the gamma, you can make an image either darker or brighter. These luminance changes are the result of gamma integration and the calculations below.

Gamma versus Gamut

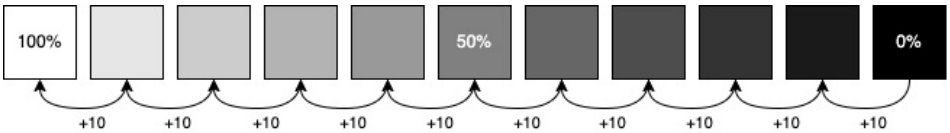
Do not confuse **gamma** with **gamut** :

- The gamma affects, among other things, the contrast of the image and the values such as 2.2, 2.4, 2.6... will be used to express it.
- A Gamut is a set of colors within a colorspace.

In a linear system, if we want to go from white to black, there will be a gradual and straight progression. For example, let's define that the black value as 0, the white as 1 and the gray as 0.5. We will have perfectly balanced intermediate steps which will form a perfectly straight line.



Each color will be equidistant from the previous and the next color :

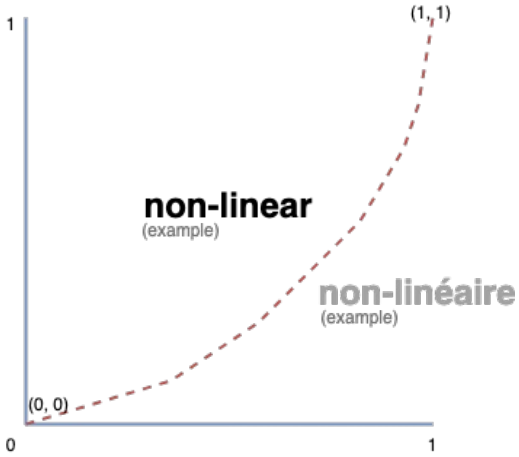


In our example, with 10 values from absolute black to absolute white, each step increases by the same value (here, +10, but you can have another value).

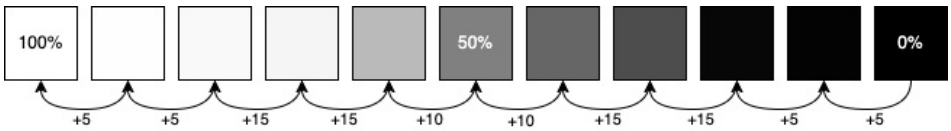
For example, a camera operates using a linear system. Each photon is captured by a photosite on the sensor. If there is less light, the sensor converts fewer details. On the contrary, if there is more light, the sensor converts it with more details.

The human eye doesn't work the same way : An eye adapts to its environment and tries to counterbalance the lack of light, the darkness (low light, shadows) and the intense brightness (highlights). The eye is more sensitive to changes in luminosity than the changes in color. The eye can adjust the ambient lighting color to adapt certain color perceptions (such as white) in relation to its environment [1] : the eye perceives light non-uniformly, its response is non-linear.

A non-linear system will produce a more or less distorted curve. Here an example with a gamma using a logarithm :



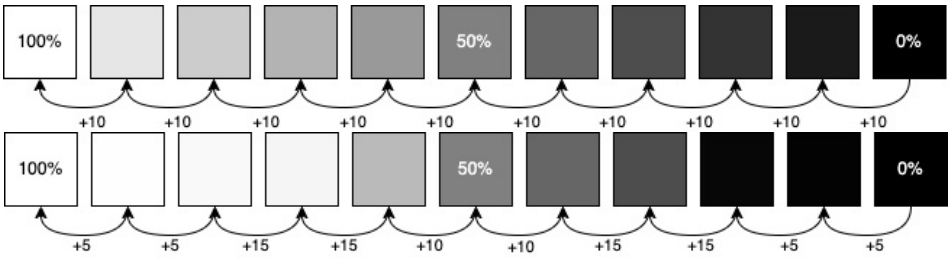
Thus, we will have this following form :



> Don't rely too much on these numbers, they aren't related to the diagram above. The values are provided only to illustrate the non-linearity, the lack of proportionality between each value :)

The transition from one value to another creates a certain curve.

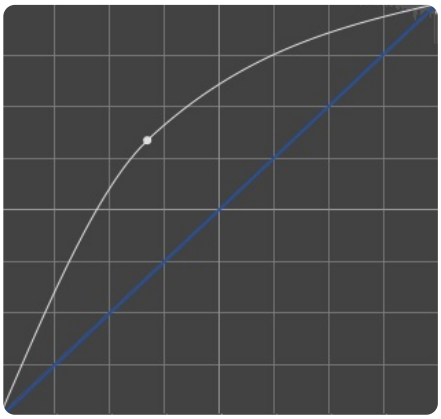
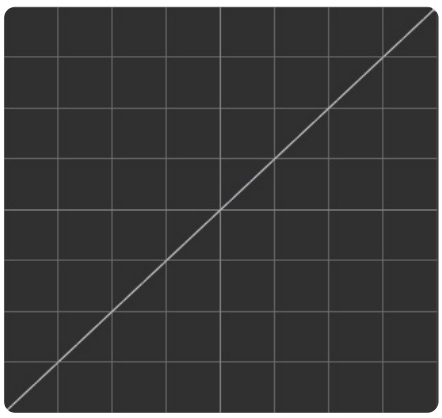
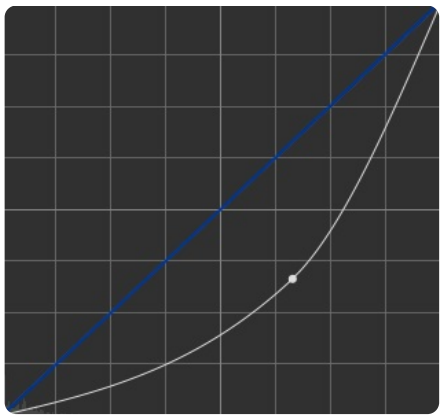
If we quickly compare both linear and non-linear example :



We observe that the whites are more white at the beginning and at the end; while blacks stay more to black.

> Note: with a real gamma, the non-linear values will differ from those in the example above.

A gamma allows an image to better match our real, non-linear visual perception. Thus, we apply a specific encoding on the image to better match its perception :

Gamma lower than 1.0	Gamma 1.0 (neutral)	Gamma upper than 1.0
		
		

A gamma curve will be more or less distorted depending on its value (2.2, 2.4, 2.6, ...)

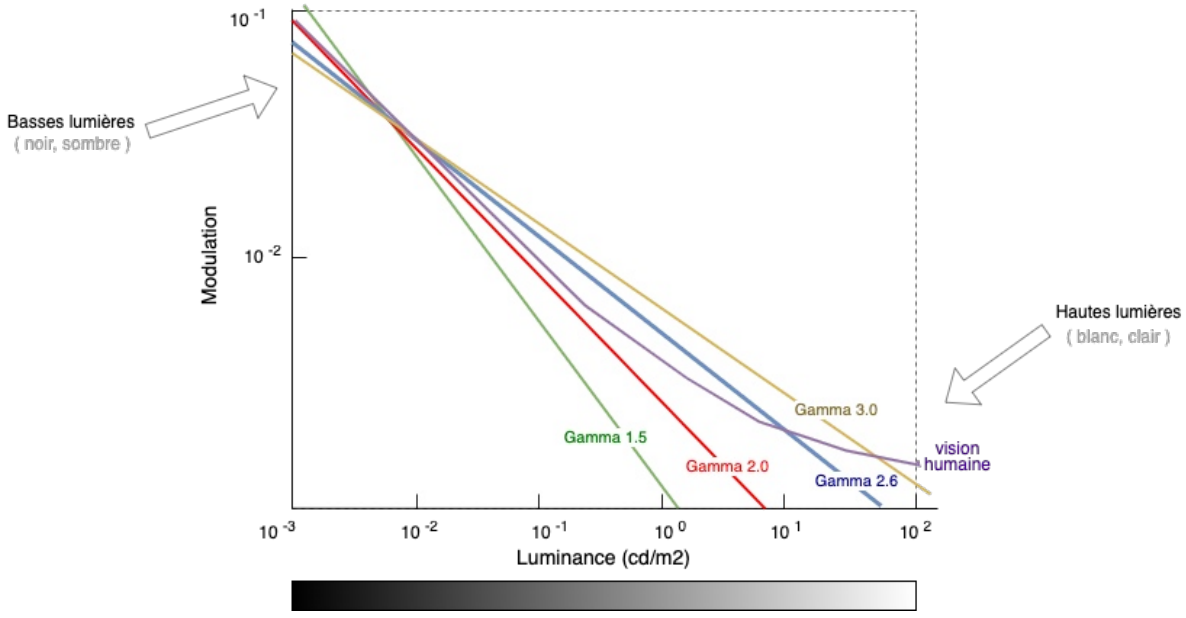
We apply a gamma according to the viewing environment : the darker the room, the higher the gamma value. The brighter the room, the lower the gamma value :

- **In theater, the gamma is 2.6** [2]
- At home, the gamma will be 2.4 or 2.2
- In an office (for example), the gamma may be 2.2

So, why we use a gamma 2.6 ? Why not a gamma 2.8 or 3.0 ?

First, the theater is a very special place with a optimal environment compared to other environnment (such as office or your home, where the light can come from anywhere). The theater is a dark place without any stray light, allowing us to accentuate the contrast of the picture.

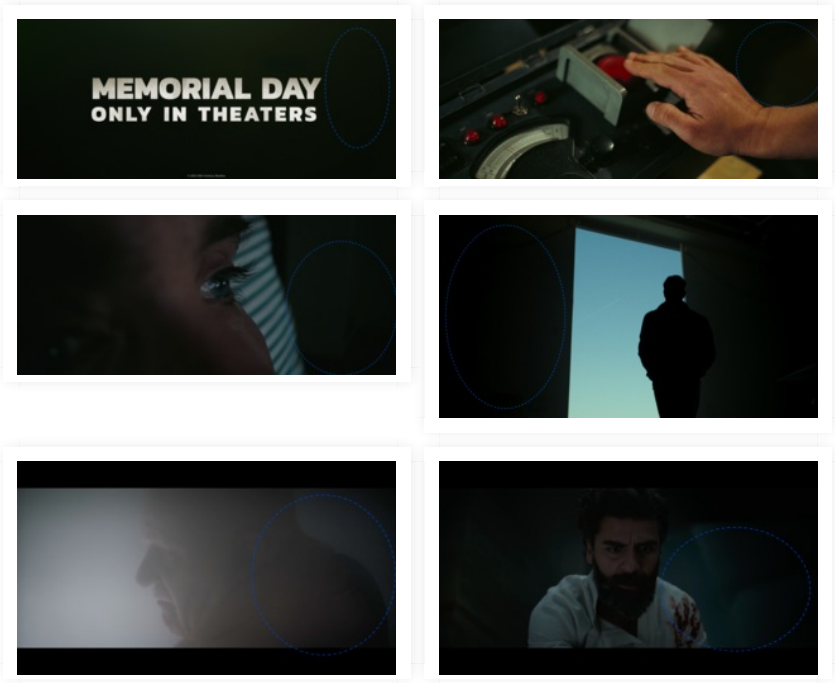
Another aspect is that a gamma of 2.6 closely matches human vision in this context of darkness. [3] :



Based on the data from the book **Color and Mastering for Digital Cinema**, the diagram shows that the gamma of 2.6 is the most closely matches human vision.

In the top-left part, we have the low light (or shadows, dark). In the bottom-right part, the highlights (white, bright). You need to work with the different value levels in a way that stays below the threshold of human visual detection, while also avoiding unnecessary waste of values.

The purple curve matches the human vision. It should stay as much as possible below to avoid banding in the picture. Color banding happens when the transition from one color to another is too abrupt. These color bands may appear depending on the bitdepth chosen or the colorspace (gamut) used, and not just because of the wrong gamma. Most of the time, you can observe banding in the TV stream or on a DVD, particulary in the darkest areas of the picture. Here are some example with color banding from trailers :



It's important to understand that applying a gamma is mainly a technical choice: a gamma allows gradients (transition from one value to another) to be less visible to the human eye. Abrupt changes in value are more visible. A gamma curve allows values below the threshold of human vision (in purple).

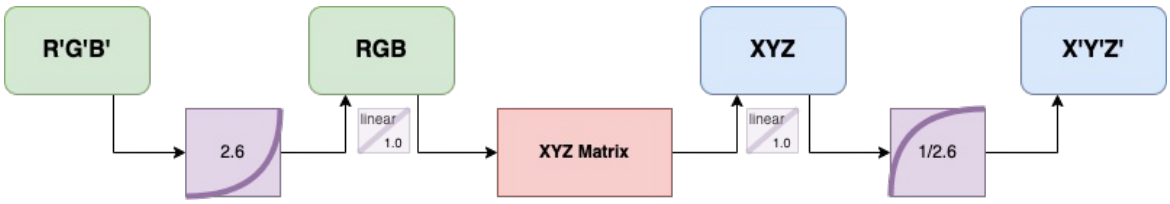
Among all feasible gamma curves, gamma 2.6 is a good compromise.

MODIFY GAMMA

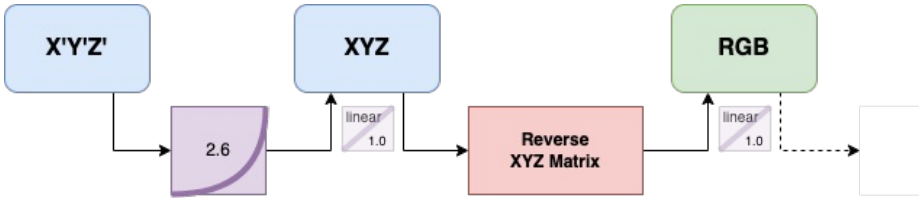
Modifying gamma means either linearizing or delinearizing, both of which require a type of calculation based on the Power law using a gamma value of `2.6` as input (must be adapted according to the input) and `1/2.6` as output (DCI).

Don't forget the use of prime symbol `'` means a gamma. Thus, a `X'Y'Z'` refers to a `XYZ` with a gamma. [4]

Here are some examples of the gamma transformation from `R'G'B'` to `X'Y'Z'` and vice-versa :



From our `R'G'B'`, we remove its 2.6 gamma curve (although it could also be a 2.2 gamma), it give us a linear `RGB` (1.0). From there, we convert to `XYZ`, then we apply a 2.6 gamma (`1/2.6`) resulting in a `XYZ` with a gamma : `X'Y'Z'`



Here, it's the reverse, we start with an `XYZ` with gamma (`X'Y'Z'`), remove its 2.6 gamma (in blue), resulting in a linear `XYZ` (without gamma, or with a 1.0 gamma). After a reverse conversion, we obtain our `RGB`. From there, we can proceed with another conversion.

In the examples, **we will work with 16-bit** to be more precise.

POW ! POW ! POW ! THAT'S THE POWER OF THE FUNK !

In our document, we will use the `pow()` function to apply the power law in our calculations.

In **mathematics**, the `pow()` is a simple equation: $f(x) = a.x^g$
The variable `g` represents our gamma value, which is either `2.6` or `1/2.6`.

In **Python** - and in other languages - you can use the `pow()` function or the double-star :

```
>>> pow(1.5, 2.6)
2.8697051264080295

>>> (1.5**2.6)
2.8697051264080295
```

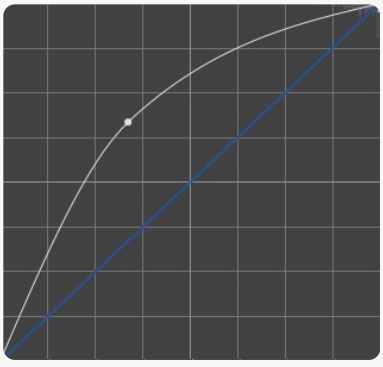
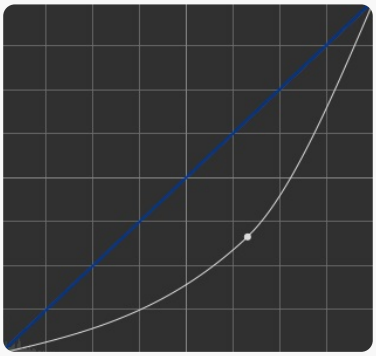
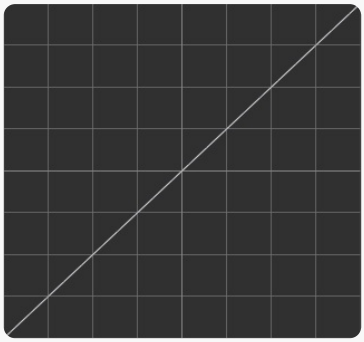
In **C & C++**, you can use the `powf()` function :

```
#include <stdio.h>
#include <math.h>

int main(void) {
    float a = 1.5;
    float g = 2.6;
    printf("%.022f\n", powf(a, g));
    return 0;
}

# gcc powfy.c -o powfy && ./powfy
2.869704961776733398437 5
```

In the rest of the document, we will only use the `pow()` function to maintain consistency with several programming languages.

REMOVE GAMMA (OR TO MAKE IT NEUTRAL)

"Removing" a gamma means returning to a gamma of 1.0 :

Gamma 0.38 (1 / 26)		Gamma 2.6		Gamma 1.0 (neutral)
	+		=	

To make a gamma neutral, you just need to :

- Apply a gamma `2.6` to a gamma `1/2.6`
- And vice-versa, Apply a gamma `1/2.6` to a gamma `2.6`

From this point, we can see the different calculations in the following paragraphs.

HOW CAN WE DETERMINE THE VALUE OF THE INPUT GAMMA ?

It's a real dilemma. By default, they is no metadata that determines the gamma value (except if the file format allows including this type of metadata). Thus, we must determine it based on its colorspace.

Level	Usage
2.2	sRGB, ...
2.35	Rec709 EBU Standard (rarely used)
2.4	Rec709 to compensate for high contrast on some display monitors, Rec2020
2.6	DCI P3

Note that even if a standard exists for a type of colorspace and defines a specific gamma value (such as the sRGB gamma normalized to 2.2), you can have RGB with a gamma of 2.6 or even 2.0: the image creator might apply a more or less contrast for various reasons (even the craziest ones :)

Thus, be sure to verify wether the input gamma matches the standard-defined gamma.

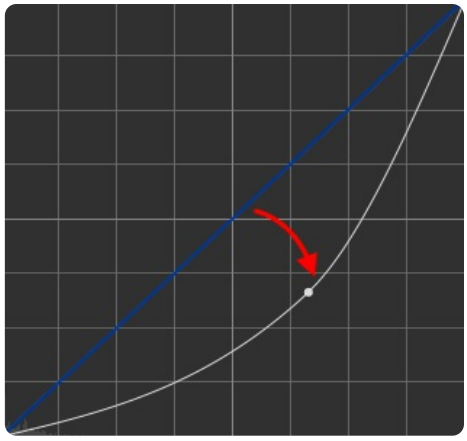
DCI GAMMAS

Quick summary of the input and output DCI gammas before continuing :

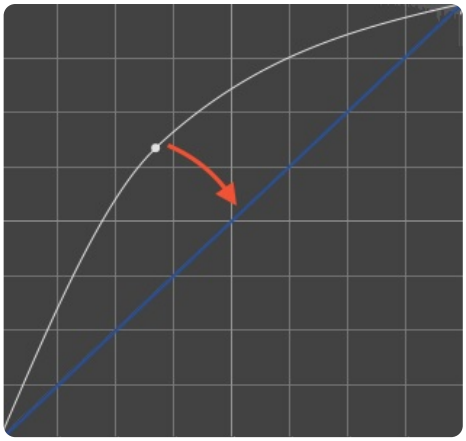
- In DCP, the gamma is `1/2.6`
- A projector removes a gamma of `1/2.6` by applying a gamma of 2.6.

APPLY A GAMMA OF `2.6`

A gamma of `2.6` may be applied in two cases :



If you have a gamma of `1.0` (neutral),
apply a gamma of `2.6` which makes your picture darker.



If you have a gamma of `1 / 2.6`,
apply a gamma of `2.6` which makes it neutral,
so linearized (For example, if you want to convert to `XYZ`)

CONVERSION OF THE WHITE COLOR USING A GAMMA 2.6

To better understand the gamma conversion, here is the first example without a [0..1] range conversion and using the `pow()` function :

```
$ python3
r = pow(0xFFFF, 2.6)           # 0xFFFF = decimal value of "65535"
3332963746773.969

g = pow(0xFFFF, 2.6)
3332963746773.969

b = pow(0xFFFF, 2.6)
3332963746773.969
```

In the white color, the red, the green and the blue are in maximal value, so `0xFFFF`.

> For our example, I'm using the hexadecimal value `0xFFFF` because I'm used to working directly with the values

that I can see in a hexadecimal viewer. But it's better to work with decimal values - 65535 in our case.

Note that we are working with raw values (hexa or decimal): Using these kind of values may cause some issues: The floating-point values produced by `pow()` output are large, excessive and not useful in a mathematical workflow, and they may cause precision issues. That's why it's better to normalize all of these values to a safer base by using a [0..1] range. For reminder, putting a value into the [0..1] range means converting any decimal value into a value between 0.0 and 1.0.

Now, let's see an example with the [0..1] range conversion. To do this, we divide your colorimetric value by its maximum value for the chosen bitdepth (here 16 bits, so `0xFFFF`) :

```
$ python3
VALUE_MAX_16BITS = 0xFFFF    # 65535
r = pow(0xFFFF / VALUE_MAX_16BITS, 2.6)
1.0

g = pow(0xFFFF / VALUE_MAX_16BITS, 2.6)
1.0

b = pow(0xFFFF / VALUE_MAX_16BITS, 2.6)
1.0
```

It's clearer with the [0..1] range : the minimum and maximum values produced by `pow()` will always be 0.0 (min) and 1.0 (max) respectively.

Note that with a 12-bit bitdepth, we will get (almost) the same result :

```
# 16-bit gamma conversion
>>> pow(0x7FFF / 0xFFFF, 2.6)       # 0x7FFF = 32767  /  0xFFFF = 65535
0.16493194524645807

# 12-bit gamma conversion
>>> pow(0x7FF / 0xFFF, 2.6)
0.16483378645422764
```

The difference between 16-bit and 12-bit is totally normal, we don't have the same precision, so the power law will be different in the end. However, we observe that both results are 0.164.

CONVERSION OF THE DARK-BLUE COLOR USING A GAMMA 2.6

Our first example without the [0..1] range conversion :

```
r = pow(0x1212, 2.6)       # decimal value: 4626 (out of 65535)
3384812638.485899

g = pow(0x3434, 2.6)       # decimal value: 13364 (out of 65535)
53386913590.93187

b = pow(0x5656, 2.6)       # decimal value: 22102 (out of 65535)
197479534076.03644
```

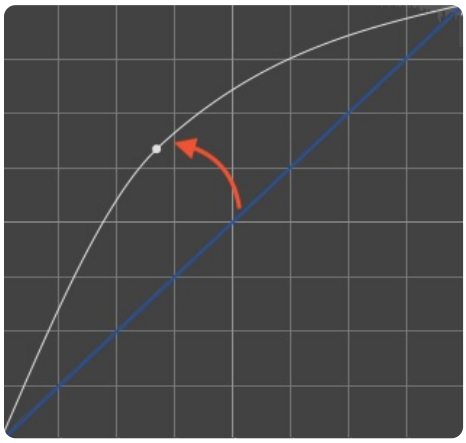
Using the [0..1] range conversion :

```
VALUE_MAX_16BITS = 0xFFFF    # 65535
r = pow(0x1212 / VALUE_MAX_16BITS, 2.6)
0.0010155563923436356

g = pow(0x3434 / VALUE_MAX_16BITS, 2.6)
0.016017850071908507

b = pow(0x5656 / VALUE_MAX_16BITS, 2.6)
0.05925042967154389
```

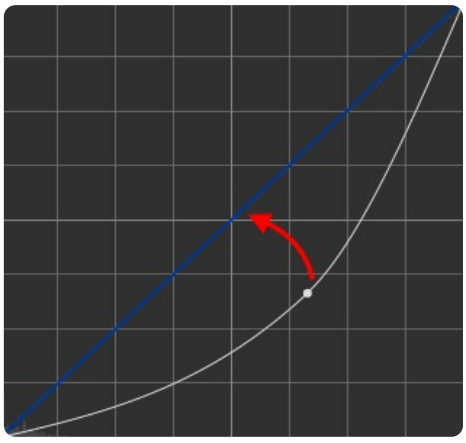
These values are linearized.

APPLY A GAMMA OF `1/2.6`

A gamma of `1/2.6` may be applied in two cases :



If you have a gamma of `1.0` (neutral),
apply a gamma of `1 / 2.6` will produce a brighter picture.



If you have a gamma of `2.6`,
apply a gamma of `1 / 2.6` will make it neutral,
so linearized (For example, if you want to convert to `XYZ`)

EXAMPLE OF `1/2.6` CONVERSION

We take the linearized values of white :

Without [0..1] range conversion :

```
r = pow(3332963746773.969, 1/2.6)
65534.99999999996
>>> hex(int(round(65534.99999999996)))
'0xffff'

g = pow(3332963746773.969, 1/2.6)
65534.99999999996

b = pow(3332963746773.969, 1/2.6)
65534.99999999996
```

With the white color, we observe that our output value is nearly the maximum possible value for 16-bit bitdepth (65535). If we apply a `round()` and then `hex()`, the result is exactly `0xFFFF`.

The same example using the [0..1] range conversion with the dark-blue color ■ :

```
>> VALUE_MAX_16BITS = 0xFFFF    # 65535

# (R)
>> pow(0.0010155563923436356, 1/2.6) * VALUE_MAX_16BITS
4626.000000000001

>>> hex(4626)
'0x1212'

# (G)
>> pow(0.016017850071908507, 1/2.6) * VALUE_MAX_16BITS
13364.000000000002
>>> hex(13364)
'0x3434'

# (B)
>> pow(0.05925042967154389, 1/2.6) * VALUE_MAX_16BITS
22102.0

>> hex(22102)
'0x5656'
```

The results are `0x1212`, `0x3434` and `0x5656`, just like before.

FILES AND ASSETS

There are the differents files, tools, codes and assets used in this chapter :

- **Tools and codes** :

- Conversion: TIFF 16-bits to 8-bits : conversion_16bits-to-8bits.py
- Conversion: TIFF 16-bits to 16-bits DCDM (12-bits into 16-bits) : conversion_16bits-to-16bits-DCDM.py
- Conversion: RGB → XYZ (16-bits) (with Gamma) : conversion_rgb2xyz.py
- Conversion: XYZ → RGB (16-bits) (with Gamma) : conversion_xyz2rgb.py
- Conversion: RGB → XYZ (without Gamma) : conversion_rgb2xyz-without-gamma.py
- Conversion: RGB → XYZ (without [0..1] range conversion) : conversion_rgb2xyz-without-conv-bitdepth.py
- Tests out-of-bound : tests_out-of-bound.py
- Tests conversions drifts : tests_drifts.py

- **Assets** :
  - RGB 16-bits : rgb-16bits.tif
  - XYZ 16-bits : xyz-16bits.tif
  - XYZ 16-bits (without Gamma) : xyz-16bits-without-gamma.tif
  - RGB 8-bits : rgb-8bits.tif
  - XYZ 8-bits : xyz-8bits.tif
  - RGB 16-bits DCDM (12-bits into 16-bits) : rgb-16bits-DCDM.tif

REFERENCES

Ressources :

- **Color and Mastering for Digital Cinema** (Glenn Kennel, Edition Focal Press)
- Gamma FAQ - Frequently Asked Questions about Gamma (Charles Poynton)

Autres ressources :

- Understanding Gamma + other Transfer Functions
- Gamma Correction
- Understanding Gamma Correction
- BenQ Laboratoire : Qu'est-ce que le gamma ? (in French)

NOTES

1. This is precisely why, if you're standing outside a house and looking into a room, you might see warm colors (like yellow or orange). But once you go inside, after a short while, you'll no longer perceive those warm tones as strongly - your eyes will have adapted. To learn more, see the article on visual adaptation ↩

2. References to the application of a 2.6 gamma in the transformation process : ↩

   - *« Color conversion from R'G'B' to X'Y'Z' requires a three-step process which involves **linearizing the color-corrected R'G'B' signals (by applying a 2.6 gamma function)** »* -- SMPTE RP-431-2-2011 - DCinema Quality Reference Projector and Environement - Chapter « Color Conversion to XYZ »
   - *« The digital files were linearized **(applying a gamma of 2.6)**, then a 3x3 matrix was applied to convert RGB to XYZ, **followed by application of the 1 / 2.6 gamma function**. The finished color-corrected files were stored as 12-bit X'Y'Z' data in 16-bit TIFF files. »* -- Color and Mastering for Digital Cinema (Glenn Kennel)
   - *« First, the R'G'B' data is linearized by **applying a simple gamma 2.6 transfer function** :* $R = (R / 4095)^{2.6}$ *»* -- SMPTE RP-431-2-2011 - DCinema Quality Reference Projector and Environment - Chapter « Color Conversion to XYZ »

3. And also to minimize the number of bits required for encoding, avoid contouring artifacts in image rendering, and achieve a better distribution of value levels within a limited encoding space (here, 12 bits). ↩

4. The X'Y'Z' notation show that the XYZ includes its transfer function which is - normally - only the gamma. However, in some documents, the white point normalization is included, and others suggest that the transfer function encompasses the entire equation $int(4095(L * X / 52.37)^{(1/2.6)})$, thus combining the white point normalization, the gamma and the bitdepth. ↩